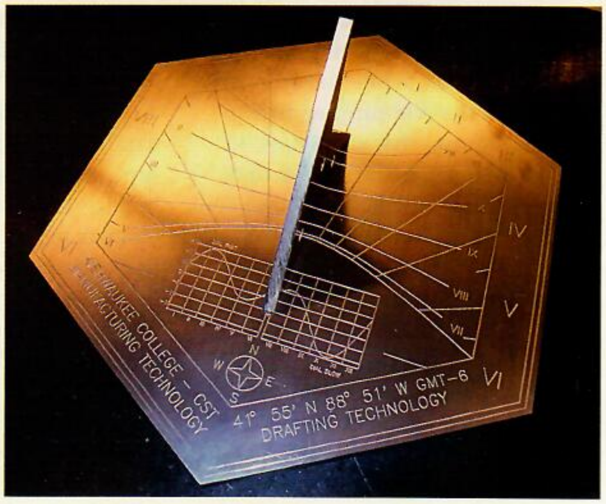
A great project that combines CAD savvy with the development of math skills



It's High Time to Make Sundials!

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LIKE so many vocational-technical educators today, I struggle to develop lessons that incorporate comprehensive design projects. My first challenge is to introduce design process with a problem that forces students to think unconventionally while coming to appreciate industrial production methods and materials.

My second challenge is to have students apply the principles of our general education curriculum in math and science, areas the solutions to design problems require.

The problem I selected for my students last year involved a common form of garden-variety sundial known as the *horizontal sundial*. Students must position this dial's pointer, called a *gnomon*, parallel to the earth's axis so that its angle of inclination identically replicates on the horizontal dial plate its geographical location away from the equator.

Since the time scales of horizontal dials placed anywhere other than on the equator (in the equatorial plane) are not equiangular, students must calculate the angles between the time marks using constructive geometry; trigonometric formulae; or downloadable, online sundial-calculation programs.

Once the horizontal sundial was designed in CAD, we could export the file to our CNC lab where the CNC router milled in the dial face with markings of what is called the sundial's "furniture," including hour and Zodiac lines, the Equation of Time graph, and the geographic location of where we meant to locate the dial.

Sundial Theory

The study of sundials, or *gnomonics* as it is sometimes called, can provide students a good understanding of some fundamental astronomical principles and practical applications of geometry, algebra, and trigonometry.

Since clock-accurate sundials are designed for specific locations on the face of the Earth, given in latitudes and longitudes, a good first step in planning a sundial design is to establish your latitude and longitude. The internet offers a wealth of automated GIS mapping services to accomplish this by simply typing in your city and state or just your zip code. I like the U.S. Census Bureau's *Tiger Map* found at **http://tiger.census.gov/cgi-bin/mapbrowse-tbl**.

As our Earth rotates on its axis, our sun appears to move uniformly across the sky. If a rod is placed parallel to the Earth's axis, its shadow will move uniformly around itself.

In other words, as the sun moves through an arc of 15° in the sky in one hour, the shadows it produces move at the same rate. Many sundials are based on this principle, as well as some equatorially mounted telescopes.

Because our Earth's distance from the sun varies throughout the year, and because the equator inclines by 23.5° to its orbit, *apparent solar time*, time told by the sun, differs from *mean solar time* (also known as Greenwich Mean Time, or GMT), time kept by our clocks. Because of this, sun and clock time, when compared with each other, may differ by as much as 15 minutes.

The variation between apparent and mean solar time is called "the equation of time," which allows us to mathematically correct the time read on a sundial as a function of a given date. The website **http://sundials.org/faq/tips.htm** shows this equation's data, often graphed.

Since graphs can be illustrated two different ways that apparently contradict each other, they can be very confusing. One type shows the

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sun running five minutes fast on a given date, so that we must subtract from the sundial, while the other type shows the dial running five minutes slow on that same date, so that we must add to the sundial time.

For example, those that show negative values for, say, August 7 show the *sun's* relationship to the *dial*. Those that show positive values on this same date show the *dial's* relationship to the *sun*.

Another correction made to sundials accounts for the longitude of their geographic location. For example, Chicago lags six hours behind Greenwich Mean Time (-6 GMT), so that noon in Greenwich is 6 a.m. in Chicago.

This is because Chicago is six hours of longitude—or 90°—west of Greenwich. Thus, based on your sundial's degree of separation from a desired angular time meridian, you can incorporate longitudinal corrections in your dial's design.

Since Chicago's longitude is 87.5°, a clock-accurate sundial will read 2.5° fast in Chicago because it is east of the Central Time Zone Meridian. To correct for this difference, you would rotate the hour lines of the dial 2.5° clockwise.

For more detailed information on sundial terms and functionality, see this outstanding internet page: **http://sundials.org/faq/glossary/glossary.htm**.

Gnomon Construction

Sundial construction depends upon principles of constructive, or gnomonic, geometry. A horizontal dial designed for Chicago's latitude radiates as a 42° ellipse.

The gnomon's 42° angle makes the sun's circular orbit within the equatorial plane lie as an ellipse on a projected horizontal plane. Ancient Greek astronomers and mathematicians made this calculation by experimenting with the observed angles and movement of the sun in conic sections.

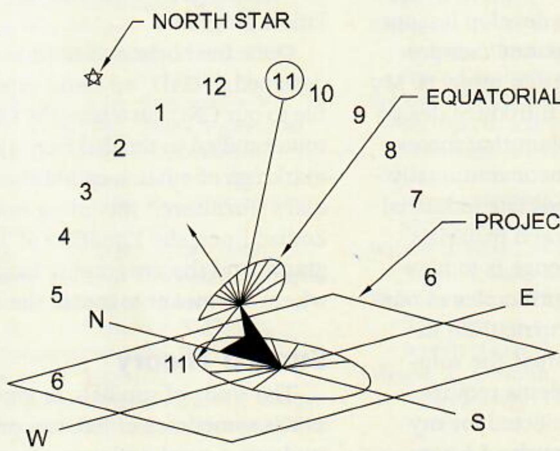
Their dial designs and graphic experiments using constructive geometry revealed the Earth's size and shape. From this knowledge, they went on to determine the size of the moon and its distance from Earth and the fact that both it and the Earth orbit the Sun.

Sundials placed at the equator parallel the apparent path of the sun, and their gnomon's shadows project direct analogs of the sun's circular motion. In other words, one hour—or 15°—of the Earth's orbit of the sun would yield 15° of movement of its shadow.

Horizontal dials are designed for use away from the equatorial plane where the sun's motion generates shadows that sweep in wide arcs early and late in the day, and narrow arcs at midday. These shadows radiate on elliptical, rather than circular, arcs.

The arrow pointing toward the North Celestial Pole in Fig. 1 is close to the axis about which the sun rotates. To make a well-designed horizontal sundial tell time, its gnomon must run along this axis.

Fig. 1



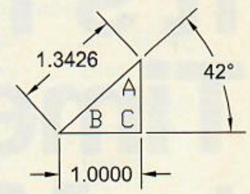
As the sun rotates around the gnomon, it casts a shadow onto the sundial face where hour lines are marked. Both the gnomon's angle and the hour lines' position depend on the latitude of the sundial's placement.

On a horizontal dial, hour lines are constructed about an ellipse that equals the degree of latitude at which the dial is located. This is called *ellipse radiation*.

For this example we use a 42° ellipse to determine the hour lines' radiation on a horizontal dial at 42° latitude. To construct such an ellipse, we need concern ourselves only with the relative lengths of the ellipse's major and minor axes. Since we require only the radials and not the ellipse's perimeter, exact size is unnecessary.

One way to constructively deduce these lengths is by drawing a gnomonic triangle for the selected latitude (Fig. 2). Construct right triangle *ABC* such that $<ABC$ equals the desired ellipse for a given latitude, in this case, 42°.

Fig. 2



Where $\overline{BC}$ represents the dial face ellipse's minor axis, $\overline{AB}$ is used to determine the length of a 42° ellipse's major axis. Those with a trigonometry background and a scientific calculator can also use the formulas 1/sin 42 to derive the exact proportions.

Dial Plate Construction

The gnomonic triangle holds all the information you need to draw a sundial plate and its hour lines by the elliptical coordinate method. See Fig. 3.

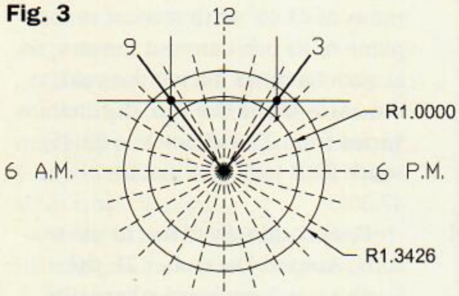
1. Using the data from the previous gnomon example, draw two concentric circles such that the inner circle has a radius of 1 and the outer circle has a radius 1.3426.

2. Draw radials set at 15° intervals. These intervals equal the hour lines of an equatorial dial and are the basis for drawing the corresponding hour lines on the ellipse of the horizontal dial. An array of 24 radials can be made with the understanding that you will not use all of them in the dial.

3. Choose two opposing radials and draw a horizontal line through their points of intersection with the outside circle.

4. Draw vertical lines from where these same two radials

Fig. 3



intersect the small circle.

5. The point where these two construction lines intersect marks two opposing points on the perimeter of the constructed ellipse; i.e., for nine o'clock and three o'clock (Fig. 4).

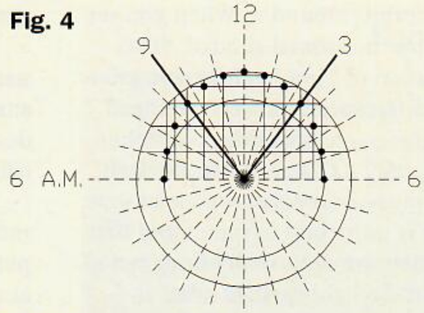
6. Repeat these constructions for each radial. The series of points generated will outline a 42° ellipse. It is unnecessary to draw the bottom half of the ellipse.

Draw revised hour lines through the respective intersection points made by the construction grid that outlines the ellipse, such as at the nine o'clock and three o'clock positions illustrated.

Longitude Adjustment

To increase the accuracy of the sundial, adjust for longitude by rotat-

Fig. 4



ing its hour lines. Since our sample dial's longitude is east of a time meridian (Central Time Zone), we can rotate the radials east, or counterclockwise.

Chicago, with a longitude of approximately 87.5°, lies 2.5° east of the Central Time Zone's meridian. Therefore, the hour lines should be rotated 2.5° clockwise. The constructed ellipse remains oriented north to south such that the major and minor axes no longer coincide with hour lines (Fig. 5).

Gnomon Placement

Make your gnomon to the previously illustrated size. Gnomons can be very decorative and functional beyond telling just the clock hour of the sun.

Some gnomons have notches on their hypotenuse whereby the notch's shadow aligns with Zodiac lines on the dial face. These Zodiac lines indicate the months from December, having the longest shadow, to June having the shortest shadow, closest to the origin of the gnomon.

You can calculate Zodiac dial furniture on a number of online sources and save them as bitmap files for tracing in CAD using Microsoft's Object Linking and Embedding feature.

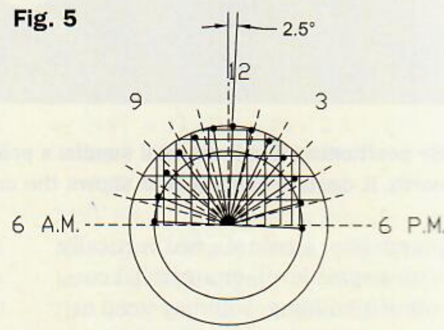
Locate the gnomon at the dial's origin where the hour line radials converge. Point the gnomon towards true, not magnetic, north. That is, align it with the major diameter of the constructed ellipse, which may or may not be 12 o'clock on the dial.

Finding True North

True north, or the North Celestial Pole, is the point in the sky about which all stars seen from the northern hemisphere seem to rotate. The North Star, also called Polaris, is located exactly at this point in the sky.

Since magnetic north and true north rarely coincide in the world, a variety of methods exist to locate

Fig. 5



this point. Using Chicago as our example, we find that it sits at -2.5° in magnetic declination.

This means that when the compass points north, true north is 2.5° east of magnetic north. So how do we field position our dial?

1. Use a compass. Since compasses point to magnetic north, you must correct for the magnetic deviation from true north by positioning the dial 2.5° to the right (east) of magnetic north.

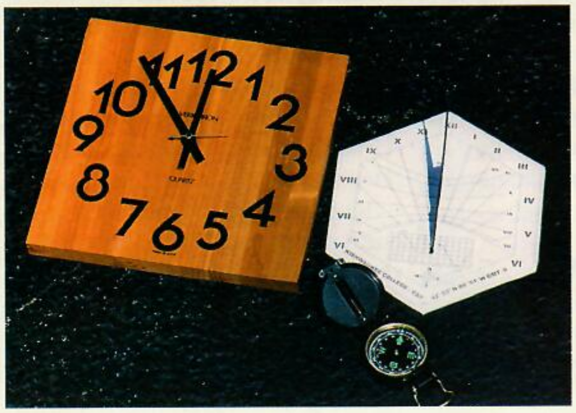
2. Mark a shadow—the shadow cast by a true vertical object—at apparent solar time, noon. To make a true vertical object, use a plumb line, a pole aligned vertically with a spirit level, or a vertical corner of a building. You may need to experiment to both get a good shadow and find a reliable way to mark it at the instant of local noon. Local noon is calculated by two variables.

a. The sun travels 15° westwards in one hour—that is, 1° westwards every four minutes, or about 950 land feet per second. In our example, Chicago lies 2.5° east of the Central Mean Time zone, or 10 mean solar time (clock) minutes before the sun reaches this meridian. Your watch must be correctly set for your time zone to consider this variable.

b. As measured by our watches, the sun runs fast or slow, by amounts detailed in "The Equation of Time" (available in the online version of this project at **www.techdirections.com/html/projects.html**) as the Earth irregularly orbits around it. When you set up your horizontal sundial, "The Equation of Time" will let you know when the sun is directly overhead for any given day of the year when you apply its values to your watch, as a plus-or-minus tolerance of time.

It is important to remember that sundials are not usually designed to report daylight savings time, although their designs may be modified to do so.

3. Use the method of equal altitudes. This requires a sunny day and a level board with a true vertical nail or stick. Draw a concentric circle around the vertical stick's base.



By positioning this horizontal sundial's pointer to true north, it casts a shadow that shows the correct time.

The shadow's position tip is noted on the circle in a morning hour, such as 11 o'clock, and in an equivalent opposite afternoon hour, such as one o'clock. Join the points with a line, then bisect this line with a second line drawn back to the vertical stylus to determine true north.

Or, mark out points along the shadow's track around noon and try to determine where the absolute shortest shadow to the vertical stylus lies. This also will indicate true north.

A good resource for determining your magnetic declination from true north is found at **www.thecompassstore.com/decvar.html**.

If, like me, you are a visual learner, you might want to investigate another site at **www.princeton.edu ~ oa.manual/mapcompass2.shtml** first.

The Sun's Declination

We previously discussed notched gnomons that track their shadows along Zodiac lines. The sun's declination through the seasons determines the length of its shadow lines.

The shadow of a gnomon's end point, or a notch in the back of the gnomon, can indicate months of the year. Some sundials include the shadow end's paths at equinoxes and summer and winter solstices.

Some include intermediate declinations as well. These curves assume irregular curves and are very decorative. They are part of what is called a sundial's "furniture."

The Earth's equator tilts a maximum of 23.45° with respect to the plane of its orbit around the sun. So at various times during the year, as the earth orbits the sun, declination varies from 23.45° north to 23.45° south for a total declination of 47.30°.

Declination gives rise to the seasons. Around December 21, the Earth's northern hemisphere tilts 23.45° away from the sun (minus value for the north), which is the northern hemisphere's winter solstice and the southern hemisphere's summer solstice.

Around June 21, the southern hemisphere tilts 23.45° away from the sun (positive value for the north), which is the northern hemisphere's summer solstice and the southern hemisphere's winter solstice.

On March 21 and September 21—the fall and spring equinoxes—the sun passes directly over the equator and the value declination is 0°.

Calculating Sun Declination:

$d = 23.45 \times \sin[360 / 365 \times (284 + N)]$

Where

d = declination

N = day number (e.g., January 1 = day 1)

Using our Chicago example, the sun's declination on August 7, 2001:

$d = 23.45 \times \sin[360 / 365 \times (284 + 199)]$

$d = 23.45 \times \sin[0.986 \times 483]$

$d = 23.45 \times \sin 476.238$

$d = 23.45 \times 0.89696535458805466$

$d = +21.03°$

Reading Time on the Sundial

Mean Solar Time (MST), or "clock time," is the artificial time we use every day to standardize our time measurements. It allows people in different locations to use the same time or to easily convert time from one location to another.

Apparent Solar Time (AST) is the time according to the position of the sun in the sky relative to one specific location on the ground. In solar time in the northern hemisphere, the sun is always due south at exactly noon.

This means that, although your clock time may be the same, someone only a few miles east or west of you will realize a slightly different solar time than you.

To calculate local solar time and its related clock time, you must calculate three things:

- the relationship between the local time zone meridian and the local longitude;
- allowances made for any applicable daylight savings time; and
- the Earth's slightly irregular motion around our sun, corrected by using "The Equation of Time" graph.

Calculating Apparent Solar Time:

AST = MST + [:04 × (LSTM - LL)] + ET

Where

AST = Apparent Solar Time, or the time the sun's shadow casts on the dial

MST = Mean Solar Time, or standard clock time, adjusted for daylight savings time, if necessary

:04 = Minutes per degree of variation

LSTM = Local Standard Time Meridian, measured in degrees, running through the time zone's center

LL = Local Longitude, measured in degrees relative to the standard meridians (+ / - for west or east, respectively)

ET = "The Equation of Time" allowance plus or minus in minutes (sun fast = dial slow)

Using our Chicago example one last time:

AST = 12:00 - 1:00 + [:04 × (90 - 87.5)] - :05

AST = 11:00 + [:04 × 2.5] - :05

AST = 11:00 + :10 - :05

AST = 11:05 on the sundial at MST noon. . . But wait a minute—we already made a longitude correction in our dial's design. . . Better make that 10:55 on the old radial dial!

Sundial Resources

Organizations

The North American Sundial Society **http://sundials.org/**

Sundials on the internet **www.sundials.co.uk/**

Books

Cousins, Frank W. (1969). *Sundials: A simplified approach by means of the equatorial dial*. London: John Baker Publishers.

Curtis, Rick. (1998). *The backpacker's field manual*. New York: Random House.

Kaufmann III, William J. (1993). *Discovering the universe*. New York: W. H. Freeman.

Mayall, R. Newton, & Margaret, W. (1973). *Sundials: How to know, use, and make them*. Cambridge, MA: Sky Publishing.

Rohr, R. R. J. (1965). *Sundials: History, theory, and practice*. Toronto: University of Toronto Press.

Waugh, Albert E. (1973). *Sundials: Their theory and construction*. New York: Dover.

Sundial Software Calculators

Analemma **www.analemma.com/**

Dialist's companion **www.shadow.net/~bobt/dcomp/dcomp.htm**

Shadows **http://web.fc-net.fr/frb/sundials/defautgb.htm**

Solar calculator **www.gcstudio.com/suncalc.html**

SunAngle **http://susdesign.com/sunangle/**

SunWatch **http://perso.wanadoo.es/sunwatch3d/english/index.html**

Geographical Information

Time zones **http://liftoff.msfc.nasa.gov/academy/ROCKET_SCI/CLOCKS/TimeZones.GIF**

U.S.C.B. Tiger Map **http://tiger.census.gov/cgi-bin/mapbrowse-tbl**

Finding true north **www.thecompassstore.com/decvar.html**

Finding true north **www.princeton.edu/~oa/manual/mapcompass2.shtml**